Functional Notation Review

You will need to know functional notation well if you want to succeed in this course. Here is a quick review.

Example: Consider the function $f(x) = x^2 - 3x$. Note that

- "f" is the name of the function for later reference.
- "(x)" says that "x" is the input to the function, anything inside the parantheses next to the function name is the input. This is NOT multiplication, the parantheses indicated input.
- "$x^2 - 3x$" is the rule. It says that whatever the input is to the function, but that input in place of all the "x" locations.

Let's input several things into this particular function, just so you can see how it works:

- Input $x = 2$ into the function $f(x)$ and you get $f(2) = (2)^2 - 3(2) = 4 - 6 = -2$.
- Input $x = 10$ into the function $f(x)$ and you get $f(10) = (10)^2 - 3(10) = 100 - 30 = 70$.
- Input $x = w$ into the function $f(x)$ and you get $f(w) = w^2 - 3w$.
- Input $x = BLAH$ into the function $f(x)$ and you get $f(BLAH) = (BLAH)^2 - 3(BLAH)$.
- Input $x = t + h$ into the function $f(x)$ and you get $f(t + h) = (t + h)^2 - 3(t + h)$.

Try the function questions (we are still taking about $f(x) = x^2 - 3x$):

1. What is $f(t)$?
2. What is $f(x + 5)$?
3. What is $f(x) + 5$?
4. What is $\frac{f(x)}{x}$?
5. What is $f(x + h) - f(x)$?

Answers:

1. $f(t) = t^2 - 3t$ (the input to f is t).
2. $f(x + 5) = (x + 5)^2 - 3(x + 5)$ (the input to f is $x + 5$).
3. $f(x) + 5 = x^2 - 3x + 5$ (the input to f is x, then we add 5).
4. $\dfrac{f(x)}{x} = \dfrac{x^2 - 3x}{x}$ (the input to f is x, then we divide by x).
5. $f(x + h) - f(x) = [(x + h)^2 - 3(x + h)] - [x^2 - 3x]$ (the function is used twice, first the input to f is $x + h$ and the second input to f is x. We need to give these expressions then subtract the second from the first). Also note, I like to use brackets, [and], to separate uses of the function, these are the same as parantheses. I just use brackets because paratheneses are used elsewhere.

More Examples Consider these functions:

$g(x) = e^x$	$h(x) = x^3 - \sqrt{x}$	$p(x) = \dfrac{x+1}{x^4}$	$q(x) = \ln(x) - 7$

For these functions, try the questions below:

1. What is $g(5+h) - g(5)$?
2. What is $h(x-3)$?
3. What is $p(b) - p(1)$?
4. What is $q(x+h) - q(x)$?
5. What is $g(x+1) - 2h(x) + q(2) + p(a)$?

Answers:

1. $g(5+h) - g(5) = e^{5+h} - e^5$.
2. $h(x-3) = (x-3)^3 - \sqrt{x-3}$.
3. $p(b) - p(1) = \dfrac{b+1}{b^4} - \dfrac{1+1}{1^4}$
4. $q(x+h) - q(x) = [\ln(x+h) - 7] - [\ln(x) - 7]$
(note that $\ln(x)$ is another example of a functional notation just with the name "ln")
5. $g(x+1) - 2h(x) + q(2) + p(a) = [e^{x+1}] - 2[x^3 - \sqrt{x}] + [\ln(2) - 7] + [\dfrac{a+1}{a^4}]$.

The Most Important Example for Calculus When we are considering average rates (which lead to instantaneous rates), we will very often see

$$\text{“The average rate of change for } f(x) \text{ from } x \text{ to } x+h\text{”} = \frac{f(x+h) - f(x)}{h}.$$

We will need to be able to write this expression and simplify it. This review has been helping you practice the first step (how to write the expression), the simplification has to do with your algebra skills which you'll practice in homework and quiz section. Let's do a few more functional notation examples for this particular important expressions.

1. For the function $f(x) = x^2$, what is $\dfrac{f(x+h) - f(x)}{h}$?
2. For the function $g(x) = e^x$, what is $\dfrac{g(x+h) - g(x)}{h}$?
3. For the function $p(x) = 4x - 3x^2$, what is $\dfrac{p(x+h) - p(x)}{h}$?

Answers:

1. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x+h)^2 - x^2}{h}$.
2. $\dfrac{g(x+h) - g(x)}{h} = \dfrac{e^{x+h} - e^x}{h}$.
3. $\dfrac{p(x+h) - p(x)}{h} = \dfrac{[4(x+h) - 3(x+h)^2] - [4x - 3x^2]}{h}$.